

UNIT 3

Image Restoration

(9 Hours) — Image Processing (CACS404)

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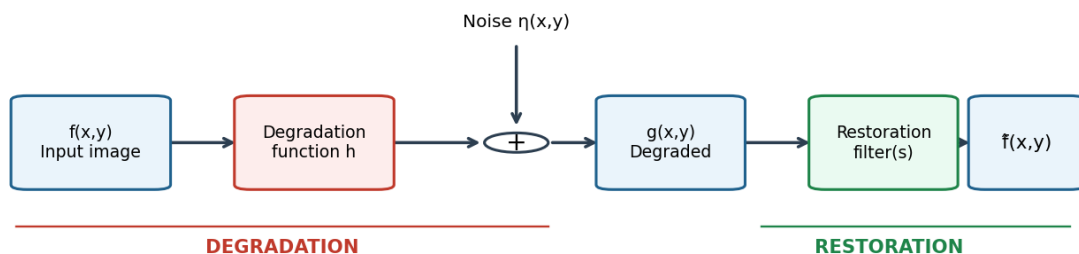
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3.1 Introduction: Restoration vs. Enhancement

Image restoration recovers an image that has been degraded, using prior knowledge of the degradation phenomenon. Restoration techniques are **objective**: they model the degradation and apply the inverse process. Enhancement, by contrast, is **subjective** — it manipulates an image to look better without any model of the degradation.

Aspect	Enhancement	Restoration
Approach	Subjective, trial and error	Objective, model-based
Goal	Better visual appearance	Reconstruct the original $f(x, y)$
Knowledge needed	None about degradation	Degradation function and noise statistics
Example	Contrast stretching, histogram equalization	Inverse / Wiener filtering, noise removal

3.2 Image Degradation Model



$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y) \Leftrightarrow G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Figure 3.1 — Model of the image degradation / restoration process

A degradation function h and additive noise $\eta(x, y)$ act on the input $f(x, y)$ to produce the degraded image $g(x, y)$:

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y) \quad (\star = \text{spatial convolution})$$

By the convolution theorem, in the frequency domain:

$$G(u, v) = H(u, v) \cdot F(u, v) + N(u, v)$$

The aim of restoration is to obtain an estimate $\hat{f}(x, y)$ as close as possible to $f(x, y)$ given g , some knowledge of h (or H) and of the statistics of η . The more we know about h and η , the closer $\hat{f}$ is to f . When h is the identity (no blur) only noise is present and spatial filters suffice; when there is blur, we need frequency-domain filters.

The degradation system is assumed **linear and position (space) invariant**, so that the response at any point depends only on the input value at that point and not on its position.

3.3 Noise Modeling

Noise arises during acquisition (sensor temperature, light level, electronic circuits) and during transmission (interference in the channel). Noise is modelled by its **probability density function (PDF)** $p(z)$ of the gray-level values and is assumed independent of the image and uncorrelated across pixels.

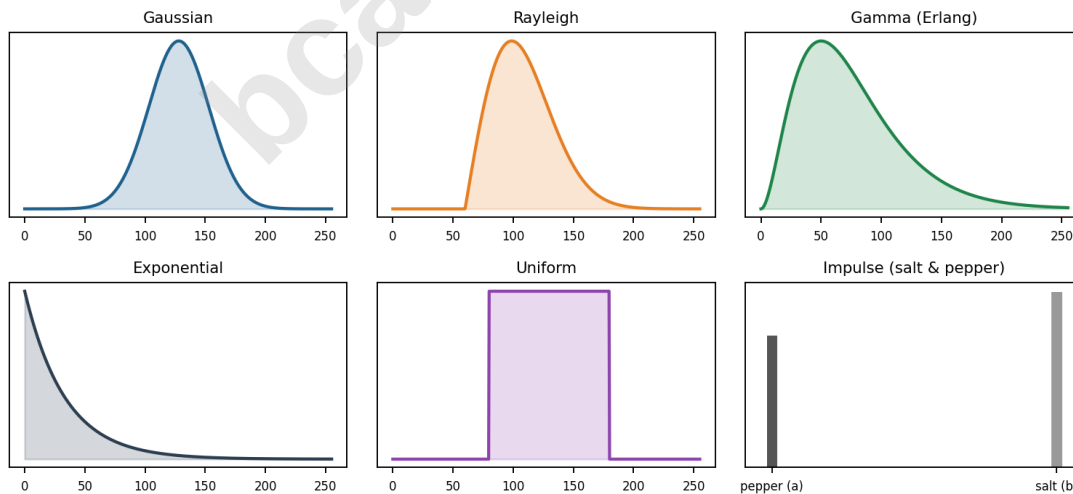


Figure 3.2 — Probability density functions of the common noise models

Noise	PDF $p(z)$	Mean / Variance	Typical source
Gaussian	$(1/\sqrt{2\pi}\sigma) e^{-(z-\mu)^2/2\sigma^2}$	μ / σ^2	Electronic circuit noise, sensor noise at low light/high temperature

Noise	PDF $p(z)$	Mean / Variance	Typical source
Rayleigh	$(2/b)(z - a) e^{-(z-a)^2/b}, z \geq a$	$a + \sqrt{(\pi b/4)} / b(4 - \pi)/4$	Range imaging; skewed histograms
Gamma (Erlang)	$a^b z^{b-1} e^{-az} / (b - 1)!, z \geq 0$	$b/a / b/a^2$	Laser imaging
Exponential	$a e^{-az}, z \geq 0$	$1/a / 1/a^2$	Laser imaging (special case $b = 1$ of gamma)
Uniform	$1/(b - a), a \leq z \leq b$	$(a + b)/2 / (b - a)^2/12$	Simulation; quantization noise
Impulse (salt-and-pepper)	P_a at $z = a$ (pepper); P_b at $z = b$ (salt); 0 otherwise	—	Faulty switching, dead pixels, transmission errors

Periodic Noise

Arises from electrical or electromechanical interference during acquisition; it is the only **spatially dependent** noise discussed here. It appears as a sinusoidal pattern and shows up as **pairs of impulses (spikes)** in the Fourier spectrum. It is removed in the frequency domain with **band-reject** or **notch filters** that block those spikes.

Estimating Noise Parameters

Choose a small, uniform (flat) region of interest (ROI) in the image; the histogram of this patch gives the shape of the noise PDF, and its sample mean $\bar{z} = \sum z_i p(z_i)$ and variance $\sigma^2 = \sum (z_i - \bar{z})^2 p(z_i)$ are used to estimate the parameters.

3.4 Restoration in the Presence of Noise Only (Spatial Filtering)

When only additive noise is present: $g(x, y) = f(x, y) + \eta(x, y)$. Spatial filters over an $m \times n$ window S_{xy} are used (g is the observed image):

Filter	Formula $f(x, y)$	Best for
Arithmetic mean	$(1/mn) \sum g(s, t)$	Gaussian / uniform noise (blurs edges)
Geometric mean	$[\prod g(s, t)]^{1/mn}$	Gaussian; loses less detail than arithmetic mean
Harmonic mean	$mn / \sum [1/g(s, t)]$	Salt noise and Gaussian; fails for pepper
Contraharmonic (order Q)	$\sum g^{Q+1} / \sum g^Q$	$Q > 0$ removes pepper; $Q < 0$ removes salt; $Q = 0 \rightarrow$ arithmetic mean; $Q = -1 \rightarrow$ harmonic mean
Median	$\text{median}\{g(s, t)\}$	Salt-and-pepper (bipolar impulse) noise

Filter	Formula $f(x, y)$	Best for
Max / Min	$\max / \min \{g(s, t)\}$	Max $\rightarrow$ pepper noise; Min $\rightarrow$ salt noise
Midpoint	$\frac{1}{2}[\max + \min]$	Gaussian and uniform noise
Alpha-trimmed mean	$(1/(mn - d)) \sum g_r(s, t)$ ($d/2$ lowest and highest values removed)	Mixed noise (impulse + Gaussian)

Note: Using the wrong sign of Q (e.g. $Q > 0$ on salt noise) can make the result worse.

3.5 Blur and Its Causes

Blur is a loss of sharpness caused by the degradation function $h(x, y)$ (also called the point spread function, PSF). Common causes:

- **Uniform linear motion blur** — relative motion between camera and object during exposure.
- **Out-of-focus (defocus) blur** — lens not properly focused; PSF is a uniform disc.
- **Atmospheric turbulence** — $H(u, v) = e^{(-k(u^2 + v^2)^{5/6})}$; k depends on turbulence strength (e.g., satellite and astronomical images).
- **Gaussian blur** — optical imperfection, scattering.

If $H(u, v)$ is known, we try to invert the process. H is found by (i) **observation** (use a small bright point or sharp edge in the image to estimate the PSF), (ii) **experimentation** (image a point source with the same equipment settings) or (iii) **mathematical modelling** (e.g., physics of motion).

3.6 Inverse Filtering

The simplest restoration approach: divide the degraded spectrum by the degradation function.

$$\mathbf{F}(u, v) = \mathbf{G}(u, v) / \mathbf{H}(u, v) = \mathbf{F}(u, v) + \mathbf{N}(u, v) / \mathbf{H}(u, v)$$

Even if H is known exactly, the second term shows the weakness: the noise $N(u, v)$ is unknown and, where $H(u, v)$ is zero or very small (typically at high frequencies), N/H becomes huge and **dominates** the estimate — the restored image is useless. Remedies: limit the filter to frequencies near the origin (cut off the division outside a radius D_0), or use a threshold: $F = G/H$ only if $|H| > \epsilon$; otherwise $F = G$ (pseudo-inverse filter). Inverse filtering works well only for noise-free or very low-noise images.

Example 3.1 — Inverse filtering

At a frequency (u, v) , $H = 0.1$, $F = 2$ and noise $N = 0.5$ (so $G = HF + N = 0.2 + 0.5 = 0.7$). Find F by inverse filtering.

Solution

$F = G / H = 0.7 / 0.1 = 7$, whereas the true F is 2. The error equals $N/H = 0.5 / 0.1 = 5$. A small noise value is amplified ten-fold because H is small.

3.6.1 Removal of Blur Caused by Uniform Linear Motion

Suppose the image $f(x, y)$ moves with time-varying displacements $x_0(t)$ and $y_0(t)$ during an exposure of duration T . The recorded image is the integral of the shifted images over the exposure:

$$g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)] dt \Rightarrow H(u, v) = \int_0^T e^{-j2\pi[u x_0(t) + v y_0(t)]} dt$$

For uniform motion with $x_0(t) = at/T$ (horizontal shift of a pixels in time T) and $y_0(t) = bt/T$:

$$H(u, v) = T / [\pi(ua + vb)] \cdot \sin[\pi(ua + vb)] \cdot e^{-j\pi(ua + vb)}$$

- Motion only in the x -direction ($b = 0$): $H(u, v) = T \cdot \sin(\pi ua) / (\pi ua) \cdot e^{-j\pi ua}$.
- H has **zeros** at $ua + vb = \text{integer} (\neq 0)$ — in the spectrum these appear as dark parallel lines perpendicular to the direction of motion. The blur length a and direction can be estimated from the spacing of these lines.
- Restoration steps: (1) estimate a, b (and T) from the image; (2) build $H(u, v)$; (3) compute $F = G/H$ (with the pseudo-inverse near zeros or use Wiener to avoid division blow-up); (4) inverse DFT.

Note: The $(\sin x)/x$ pattern means H touches zero repeatedly, so plain inverse filtering of motion blur is unstable with noise — one more reason to prefer the Wiener filter.

3.7 Wiener (Minimum Mean-Square-Error) Filtering

The Wiener filter treats both the image and the noise as **random processes** and finds the estimate $\hat{f}$ that minimises the mean-square error $e^2 = E\{(f - \hat{f})^2\}$. Assuming f and η are uncorrelated and one has zero mean, the minimum is reached in the frequency domain at:

$$\mathbf{F}(u, v) = [(1/H) \cdot |H|^2 / (|H|^2 + S_\eta/S_f)] \cdot \mathbf{G}(u, v) = [H^* / (|H|^2 + S_\eta/S_f)] \cdot \mathbf{G}(u, v)$$

where H^* is the complex conjugate of H , $S_\eta(u, v) = |N(u, v)|^2$ is the power spectrum of the noise and $S_f(u, v) = |F(u, v)|^2$ the power spectrum of the undegraded image.

- If there is **no noise** ($S_{\eta} = 0$), the Wiener filter reduces to the inverse filter.
- S_f is usually unknown. A constant K (ratio of noise-to-signal power) is substituted: $\mathbf{F} = [|\mathbf{H}|^2 / (|\mathbf{H}|^2 + K)] \cdot \mathbf{G} / \mathbf{H}$; K is adjusted interactively for the best visual result.
- Where $|\mathbf{H}|$ is large the filter $\approx 1/\mathbf{H}$ (inverts the blur); where $|\mathbf{H}|$ is small the filter $\rightarrow 0$ (suppresses noise) — a built-in compromise.

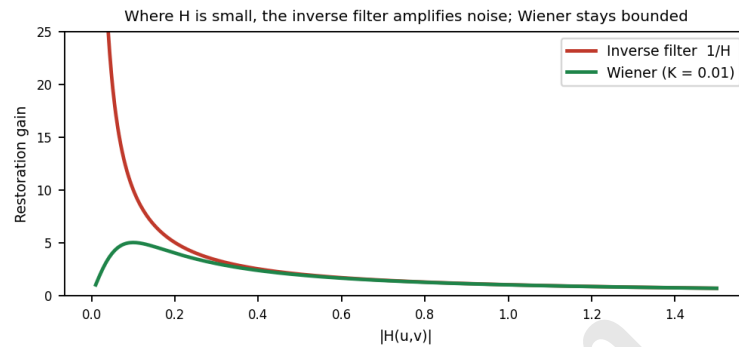


Figure 3.3 — Gain of the inverse filter vs. the Wiener filter

Example 3.2 — Wiener vs. inverse

Compare the gain applied to G by the inverse filter and by the Wiener filter with $K = 0.01$ at (a) $H = 0.1$ and (b) $H = 1$.

Solution

(a) $H = 0.1$: inverse gain $= 1/H = 10$. Wiener gain $= (1/H) \cdot |H|^2 / (|H|^2 + K) = 10 \times 0.01 / (0.01 + 0.01) = 10 \times 0.5 = 5$. The noise-sensitive frequency is attenuated by half.

(b) $H = 1$: inverse gain $= 1$. Wiener gain $= 1 \times 1 / (1 + 0.01) = 0.990 \approx 1$. Almost no loss where blur is not a problem.

Property	Inverse filter	Wiener filter
Needs	H only	H and noise/signal power spectra (or K)
Noise handling	Amplifies noise where $H \rightarrow 0$	Balances deblurring and noise smoothing
Criterion	Direct division	Minimum mean-square error
Result	Often noisy or unusable	Better visual quality; robust

3.8 Morphological Image Processing

Mathematical morphology extracts image components (boundaries, skeletons, convex hulls) that are useful for representation and description, using set theory. Images are treated as sets: for binary images, A = set of foreground (1) pixels. Operations probe the image with a small shape called the **structuring element (SE)** B , which has an origin (centre).

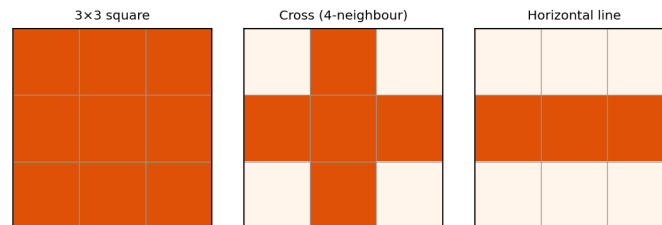


Figure 3.4 — Common structuring elements (shaded = member of B ; centre is the origin)

Reflection $B = \{w \mid w = -b, \text{ for } b \in B\}$. **Translation** $(B)z = \{c \mid c = b + z, \text{ for } b \in B\}$.

3.8.1 Erosion

$A \ominus B = \{z \mid (B)z \subseteq A\}$ — the set of all positions z where B , translated to z , fits entirely inside A .

- **Effect:** shrinks objects; removes small objects (smaller than B), thin protrusions and noise specks; enlarges holes; separates objects joined by thin bridges.
- **Implementation:** slide B over the image; output pixel = 1 only if *all* pixels under B are 1 (AND / minimum filter).

3.8.2 Dilation

$A \oplus B = \{z \mid (B)z \cap A \neq \emptyset\}$ — the set of positions where the reflected SE overlaps A in at least one element.

- **Effect:** grows or thickens objects; fills small holes, gaps and cracks; joins broken parts (e.g., broken characters in text).
- **Implementation:** output pixel = 1 if *any* pixel under B is 1 (OR / maximum filter).

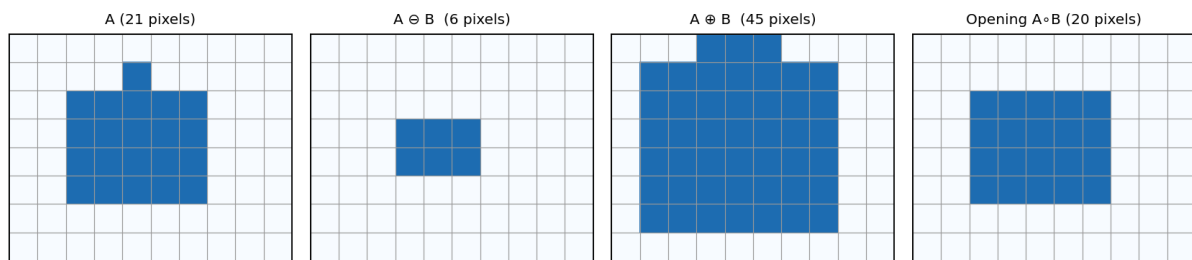


Figure 3.5 — Erosion, dilation and opening of a binary object (3×3 square SE)

Example 3.3 — Erosion and dilation

Object A is a solid block of 4 rows $\times$ 5 columns plus one extra pixel (a spur) attached above the block (21 pixels). B is a 3×3 square. Find the size of $A \ominus B$ and $A \oplus B$.

Solution

Erosion: B fits completely only where all 9 pixels lie in A. The spur has no full 3×3 neighbourhood in A, so it vanishes. The block shrinks by one pixel on each side: $(4 - 2) \times (5 - 2) = 2 \times 3 = 6$ pixels.

Dilation: the block grows by one pixel on every side: $(4 + 2) \times (5 + 2) = 6 \times 7 = 42$ pixels, and the spur adds a new row of 3 pixels above $\rightarrow 45$ pixels.

3.8.3 Opening, Closing and Other Operations

Operation	Definition	Effect
Opening	$A \circ B = (A \ominus B) \oplus B$	Erosion then dilation: smooths contours, removes small protrusions/thin bridges and small objects; keeps overall size
Closing	$A \bullet B = (A \oplus B) \ominus B$	Dilation then erosion: fills small holes and narrow gaps/channels, joins nearby objects
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Gives the object outline (internal boundary)
Hit-or-miss	$A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2)$	Detects a specific shape/pattern in a binary image
Hole filling, thinning, skeletonizing	Iterative combinations	Region filling, skeleton extraction

- **Duality:** $(A \ominus B)^c = A^c \oplus B$ — erosion of A is dilation of the background.
- Opening and closing are **idempotent**: applying them repeatedly makes no further change.
- Opening removes bright noise (salt); closing removes dark noise (pepper). Opening followed by closing is a classic noise-cleaning filter.

- For gray-scale images: erosion = local minimum filter, dilation = local maximum filter; morphological gradient = dilation – erosion.

Key Takeaways

- ✓ Restoration is model-based: $g = h \star f + \eta \Leftrightarrow G = HF + N$.
- ✓ Noise models: Gaussian, Rayleigh, gamma, exponential, uniform, impulse; periodic noise is removed with notch filters.
- ✓ Noise-only: mean, median, max/min, midpoint, alpha-trimmed, contraharmonic ($Q > 0$ pepper, $Q < 0$ salt).
- ✓ Inverse filter $F = G/H$ fails when $H \approx 0$ because N/H dominates.
- ✓ Linear motion blur: $H = T \sin(\pi(ua+vb)) / (\pi(ua+vb)) \cdot e^{(-j\pi(ua+vb))}$.
- ✓ Wiener filter minimises MSE: $F = [|H|^2 / (|H|^2 + K)] \cdot G/H$ — balances deblurring and noise suppression.
- ✓ Erosion shrinks, dilation grows; opening = erosion→dilation, closing = dilation→erosion; boundary = $A - (A \ominus B)$.

Reminder: these notes may contain errors — verify with your textbook and lecturer.

— *End of Unit 3* —