

UNIT 1

Set Theory and Real & Complex Numbers

(7 Hours) — Mathematics I (CACS104)

This unit introduces the foundational language of modern mathematics: sets, the real number system, and the complex number system. Set notation and operations underpin nearly every branch of mathematics and computer science, while the real and complex number systems provide the numerical framework used throughout the rest of this course.

1. Concept, Notation and Specification of Sets

A set is a well-defined collection of distinct objects, called elements or members. "Well-defined" means that given any object, it must be possible to decide, without ambiguity, whether that object belongs to the set or not.

Notation

Sets are usually denoted by capital letters ($A, B, C, \dots$) and elements by lowercase letters ($a, b, c, \dots$). If x is an element of set A , we write $x \in A$ (read " x belongs to A "); if x is not an element of A , we write $x \notin A$.

Methods of Specifying a Set

There are two standard ways to describe a set:

- Tabular / Roster method — elements are listed explicitly, separated by commas, inside curly braces. Example: $A = \{2, 4, 6, 8\}$
- Set-builder / Rule method — a defining property common to all elements is stated. Example: $A = \{x : x \text{ is an even natural number less than } 10\}$

Example 1.1

Write the set of vowels in the English alphabet using both the tabular and set-builder methods.

Solution

Tabular form: $V = \{a, e, i, o, u\}$

Set-builder form: $V = \{x : x \text{ is a vowel in the English alphabet}\}$

Note: The order in which elements are listed and repetition of elements do not change a set — $\{1, 2, 3\}$ and $\{3, 2, 1, 2\}$ represent the same set $\{1, 2, 3\}$.

2. Types of Sets

Type	Definition	Example
Empty (Null) Set	A set containing no elements, denoted $\emptyset$ or $\{ \}$	$\{x : x^2 = -1, x \in \mathbb{R}\} = \emptyset$
Singleton Set	A set containing exactly one element	$\{5\}$
Finite Set	A set whose elements can be counted and the counting process terminates	$\{1, 2, 3, 4\}$
Infinite Set	A set whose elements cannot be listed by a finite count	$\mathbb{N} = \{1, 2, 3, \dots\}$
Equal Sets	Two sets having exactly the same elements	$A = \{1,2,3\}, B = \{3,1,2\}$ $\Rightarrow A = B$
Equivalent Sets	Two sets having the same number of elements (same cardinality)	$A = \{a,b,c\}, B = \{1,2,3\}$
Subset	Every element of A is also an element of B, written $A \subseteq B$	$\{1,2\} \subseteq \{1,2,3\}$
Proper Subset	$A \subseteq B$ and $A \neq B$, written $A \subset B$	$\{1,2\} \subset \{1,2,3\}$
Universal Set	The set containing all objects under consideration, denoted U	$U = \{1,2,\dots,10\}$
Power Set	The set of all subsets of A, denoted $P(A)$; if $ A = n$, $ P(A) = 2^n$	$P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

3. Operations on Sets

Given sets A and B within a universal set U, four fundamental operations combine or compare them. Each is illustrated below with a Venn diagram, in which the shaded region represents the resulting set.

3.1 Union ($A \cup B$)

The union of A and B is the set of all elements that belong to A, or to B, or to both: $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

$A \cup B$ (Union)

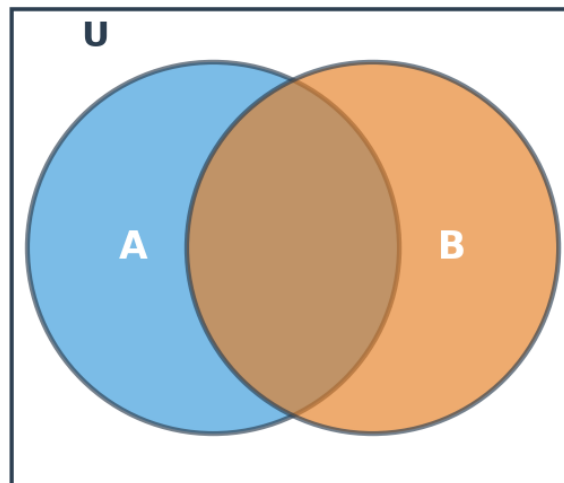


Figure 1.1 — $A \cup B$: every element in A, B, or both is included (shaded).

3.2 Intersection ($A \cap B$)

The intersection of A and B is the set of elements common to both: $A \cap B = \{x : x \in A \text{ and } x \in B\}$. If $A \cap B = \emptyset$, the sets are called disjoint.

$A \cap B$ (Intersection)

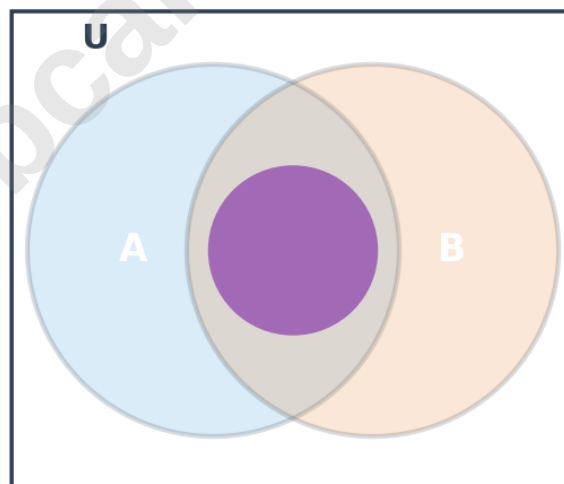


Figure 1.2 — $A \cap B$: only the overlapping region (shaded) belongs to both sets.

3.3 Difference ($A - B$ and $B - A$)

The difference $A - B$ is the set of elements in A that are not in B: $A - B = \{x : x \in A \text{ and } x \notin B\}$. In general $A - B \neq B - A$.

A – B (Difference)

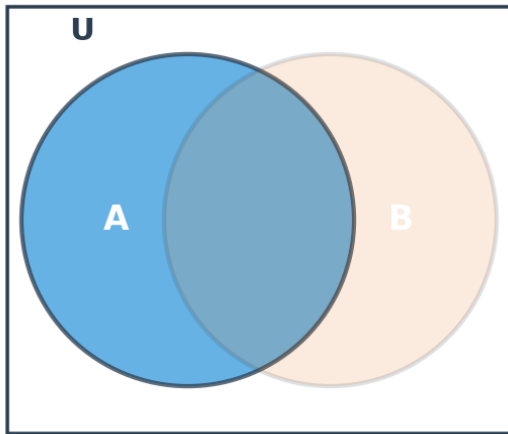


Figure 1.3 — $A - B$

B – A (Difference)

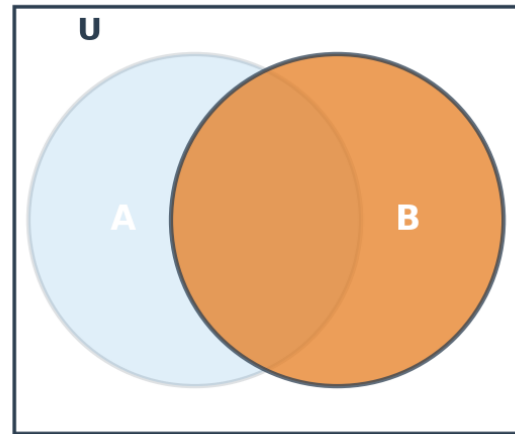


Figure 1.4 — $B - A$

3.4 Complement (A')

The complement of A , written A' (or A^c), is the set of all elements of the universal set U that are not in A : $A' = \{x : x \in U \text{ and } x \notin A\} = U - A$.

A' (Complement of A)

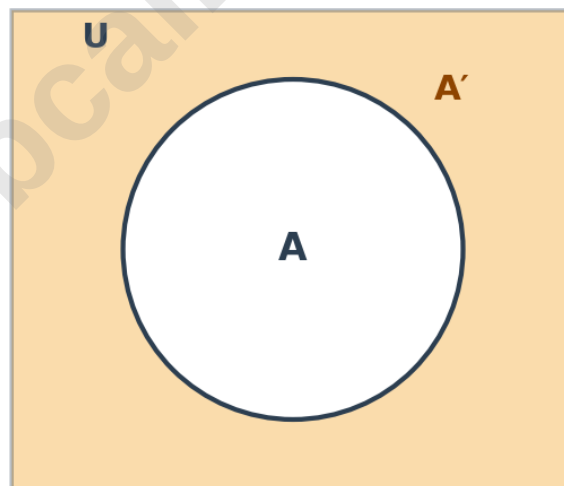


Figure 1.5 — A' : everything inside U but outside A (shaded).

Example 1.2

Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$.

Find: (a) $A \cup B$ (b) $A \cap B$ (c) $A - B$ (d) A'

Solution

(a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(b) $A \cap B = \{4, 5\}$

(c) $A - B = \{1, 2, 3\}$

(d) $A' = U - A = \{6, 7, 8, 9, 10\}$

4. Laws of Algebra of Sets

The following laws (stated without proof, as per the syllabus) hold for any sets A, B, C within a universal set U. They mirror the algebraic laws of ordinary numbers, with $\cup$ playing a role similar to addition and $\cap$ similar to multiplication.

Law	Union form	Intersection form
Idempotent	$A \cup A = A$	$A \cap A = A$
Identity	$A \cup \emptyset = A$, $A \cup U = U$	$A \cap U = A$, $A \cap \emptyset = \emptyset$
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan's Laws	$(A \cup B)' = A' \cap B'$	$(A \cap B)' = A' \cup B'$
Complement	$A \cup A' = U$	$A \cap A' = \emptyset$
Involution	$(A')' = A$	—

Note: De Morgan's Laws are especially important — they let you rewrite the complement of a union/intersection in terms of complements of the individual sets, and are widely used in logic, probability, and digital circuit design.

5. Cardinal Number of a Set

The cardinal number (or cardinality) of a finite set A, denoted $n(A)$ or $|A|$, is the number of distinct elements in A. For two finite sets A and B, the following relation — obtained directly from the Venn diagram — connects the cardinalities of A, B, their union and their intersection:

Key Formula

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

For three sets: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Example 1.3

In a class of 50 students, 30 like Mathematics, 25 like Physics, and 15 like both subjects. How many students like at least one of the two subjects? How many like neither?

Solution

Let M = students who like Mathematics, P = students who like Physics.

$$n(M) = 30, n(P) = 25, n(M \cap P) = 15$$

$$n(M \cup P) = n(M) + n(P) - n(M \cap P) = 30 + 25 - 15 = 40$$

So 40 students like at least one subject.

$$\text{Students who like neither} = \text{Total} - n(M \cup P) = 50 - 40 = 10$$

6. The Real Number System

The real number system $\mathbb{R}$ is built up from simpler number systems, each nested inside the next:

Set	Symbol	Description	Example
Natural numbers	$\mathbb{N}$	Counting numbers	1, 2, 3, ...
Whole numbers	$\mathbb{W}$	Natural numbers with 0	0, 1, 2, ...
Integers	$\mathbb{Z}$	Whole numbers and their negatives	..., -2, -1, 0, 1, 2, ...
Rational numbers	$\mathbb{Q}$	Numbers expressible as p/q , $q \neq 0$	$1/2$, -3, 0.75
Irrational numbers	$\mathbb{Q}'$	Numbers not expressible as p/q	$\sqrt{2}$, π , e
Real numbers	$\mathbb{R}$	Union of rational and irrational numbers	$\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$

Every real number corresponds to exactly one point on a number line, and every point on the number line corresponds to exactly one real number — this one-to-one correspondence is called the completeness property of $\mathbb{R}$.

6.1 Intervals

An interval is a connected subset of the real line. The main types are:

Interval	Notation	Set-builder form	Description
Closed	$[a, b]$	$\{x \in \mathbb{R} : a \leq x \leq b\}$	Both endpoints included
Open	(a, b)	$\{x \in \mathbb{R} : a < x < b\}$	Both endpoints excluded
Half-open (left)	$(a, b]$	$\{x \in \mathbb{R} : a < x \leq b\}$	a excluded, b included
Half-open (right)	$[a, b)$	$\{x \in \mathbb{R} : a \leq x < b\}$	a included, b excluded

Interval	Notation	Set-builder form	Description
Infinite	(a, ∞) , $(-\infty, b)$	$\{x : x > a\}$, $\{x : x < b\}$	Unbounded on one side

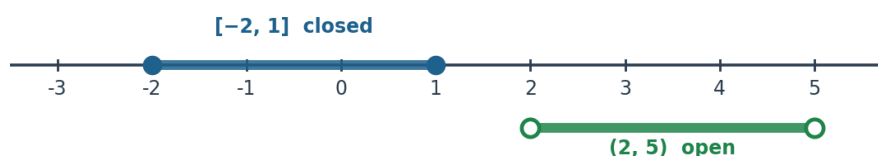


Figure 1.6 — $[-2, 1]$ shown as a closed interval (filled dots), and $(2, 5)$ shown as an open interval (hollow dots).

6.2 Absolute Value of a Real Number

The absolute value (or modulus) of a real number x , written $|x|$, is its distance from 0 on the number line, regardless of direction:

Definition

$$|x| = x, \text{ if } x \geq 0$$

$$|x| = -x, \text{ if } x < 0$$

Properties of absolute value, for real numbers x and y :

- $|x| \geq 0$, and $|x| = 0$ if and only if $x = 0$
- $|-x| = |x|$
- $|xy| = |x| \cdot |y|$
- $|x/y| = |x| / |y|$, $y \neq 0$
- $|x + y| \leq |x| + |y|$ (Triangle Inequality)

Example 1.4

Solve for x : $|x - 3| = 5$

Solution

$$|x - 3| = 5 \Rightarrow x - 3 = 5 \text{ or } x - 3 = -5$$

$$x = 8 \text{ or } x = -2$$

So the solution set is $\{-2, 8\}$.

7. Introduction to Complex Numbers

Not every polynomial equation has a solution within the real numbers — for example, $x^2 + 1 = 0$ has no real solution, since no real number squares to -1 . To resolve this, mathematicians introduced the imaginary unit i , defined by:

Definition

$i = \sqrt{-1}$, so that $i^2 = -1$

A complex number is a number of the form $z = a + bi$, where a and b are real numbers. Here, a is called the real part, written $\text{Re}(z)$, and b is called the imaginary part, written $\text{Im}(z)$. The set of all complex numbers is denoted $\mathbb{C}$. If $b = 0$, z is a real number; if $a = 0$ and $b \neq 0$, z is called a purely imaginary number.

7.1 Geometrical Representation of Complex Numbers

Just as real numbers are represented on a number line, complex numbers are represented as points on a plane called the complex plane or Argand plane. The complex number $z = a + bi$ is plotted as the point (a, b) : the horizontal axis is the real axis and the vertical axis is the imaginary axis.

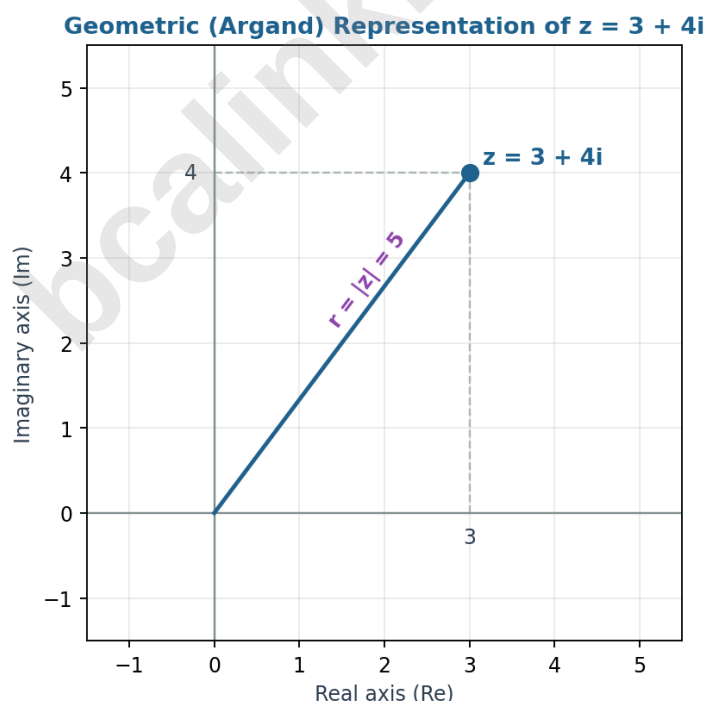


Figure 1.7 — The complex number $z = 3 + 4i$ plotted as the point $(3, 4)$ in the Argand plane. Its modulus $r = |z| = \sqrt{3^2 + 4^2} = 5$.

The distance of the point $z = a + bi$ from the origin is called the modulus of z , denoted $|z|$, and is given by $|z| = \sqrt{a^2 + b^2}$.

7.2 Simple Algebraic Properties of Complex Numbers

Let $z_1 = a + bi$ and $z_2 = c + di$ be two complex numbers.

Operation	Rule
Addition	$z_1 + z_2 = (a + c) + (b + d)i$
Multiplication	$z_1 \cdot z_2 = (ac - bd) + (ad + bc)i$
Additive Inverse	$-z_1 = -a - bi$, so that $z_1 + (-z_1) = 0$
Multiplicative Inverse	$z_1^{-1} = (a - bi) / (a^2 + b^2)$, for $z_1 \neq 0$
Absolute Value (Modulus)	$ z_1 = \sqrt{a^2 + b^2}$

Example 1.5

If $z_1 = 2 + 3i$ and $z_2 = 1 - 4i$, find (a) $z_1 + z_2$ (b) $z_1 \cdot z_2$ (c) $|z_1|$

Solution

(a) $z_1 + z_2 = (2 + 1) + (3 - 4)i = 3 - i$

(b) $z_1 \cdot z_2 = (2)(1) - (3)(-4) + [(2)(-4) + (3)(1)]i = (2 + 12) + (-8 + 3)i = 14 - 5i$

(c) $|z_1| = \sqrt{2^2 + 3^2} = \sqrt{13}$

Example 1.6

Find the multiplicative inverse of $z = 3 + 4i$.

Solution

$a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$

$z^{-1} = (a - bi)/(a^2 + b^2) = (3 - 4i)/25 = 3/25 - (4/25)i$

Note: Multiplication of complex numbers follows directly from treating i as an algebraic quantity and replacing i^2 by -1 wherever it appears — this is why the cross terms in $z_1 \cdot z_2$ combine the way they do.

Key Takeaways

- ✓ A set is a well-defined collection of distinct objects; sets can be described in tabular or set-builder form.
- ✓ The four core set operations are union, intersection, difference, and complement — each visualized with a Venn diagram.
- ✓ De Morgan's Laws relate the complement of a union/intersection to complements of the individual sets.
- ✓ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ connects cardinalities of finite sets.
- ✓ The real number system is built up as $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$, with irrational numbers filling the remaining gaps.
- ✓ Intervals describe connected subsets of $\mathbb{R}$; $|x|$ measures a real number's distance from 0 and satisfies the triangle inequality.
- ✓ A complex number $z = a + bi$ has a geometric representation as a point (a, b) in the Argand plane, with modulus $|z| = \sqrt{a^2 + b^2}$.
- ✓ Complex numbers add, multiply, and invert using $i^2 = -1$ as the only extra algebraic rule beyond ordinary arithmetic.

— *End of Unit 1* —