

UNIT 5

Inventory Control

(6 Hours) — Operations Research

Every organisation — whether a small shop, a hospital, or a large factory — keeps some amount of stock on hand. This stock, called inventory, includes raw materials, components, work-in-progress items, and finished goods. Managing inventory well is one of the most practical problems in Operations Research because it directly affects a company's cost and customer service. If you keep too little stock, you may run out and lose sales or stop production. If you keep too much stock, you tie up money and pay extra for storage, insurance, and spoilage. Inventory Control is the branch of OR that helps us decide how much to order and when to order so that total cost is kept as low as possible while still meeting demand.

5.1 Inventory — Meaning and Classification

In simple words, inventory is any idle resource that is being stored for future use. A bakery storing flour, a hospital storing medicines, and a car factory storing engine parts are all holding inventory. Inventory is classified in two common ways: by its stage in the production process, and by its nature of demand.

5.1.1 Classification by Stage in Production

Type	Meaning	Example
Raw Materials	Basic inputs purchased from suppliers, not yet processed	Steel sheets for a car plant
Work-in-Progress (WIP)	Materials that have entered production but are not finished	A half-assembled engine
Finished Goods	Completed products ready for sale or dispatch	Cars ready in the showroom
Maintenance, Repair & Operating (MRO)	Items used to support production, not part of the final product	Lubricants, spare tools, cleaning supplies

5.1.2 Classification by Nature of Demand

This is the more important classification for inventory models, because it tells us what kind of mathematics we should use.

- **Independent Demand Items:** Demand for the item comes directly from the market and does not depend on the demand for any other item. Example: finished goods sold to customers, such as a packet of biscuits. These are usually controlled using EOQ-type models, which is the focus of this unit.
- **Dependent Demand Items:** Demand for the item is derived from the demand of a parent item. Example: the demand for tyres depends directly on how many cars are being produced. These are usually controlled using Material Requirements Planning (MRP), not EOQ.

Broad Categories of Inventory Held by an Organisation

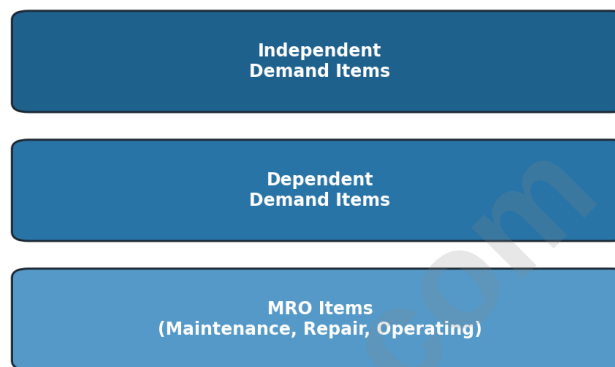


Fig. 5.1 — Broad categories of inventory commonly held by an organisation.

Note: EOQ-based models (the heart of this unit) are meant for independent demand, fairly steady-demand items such as finished goods sold continuously to customers.

5.2 Costs Associated with Inventory

Before we can decide the 'best' order quantity, we must first understand exactly which costs go up and which go down as we change that quantity. Inventory decisions are really a cost trade-off problem: ordering in large batches reduces some costs but increases others. There are four main types of cost.

Cost Type	Plain-English Meaning	Behaviour as Order Size (Q) Increases
Ordering / Set-up Cost (Co)	Cost of placing one purchase order or one production run — paperwork, transportation, machine set-up, inspection	Decreases per unit (fewer orders needed per year)
Carrying / Holding Cost (Ch)	Cost of keeping one unit in storage for one year — warehouse rent, insurance, spoilage, interest on money tied up	Increases (more average stock sitting around)

Cost Type	Plain-English Meaning	Behaviour as Order Size (Q) Increases
Shortage / Stock-out Cost (Cs)	Cost incurred when an item is demanded but not available — lost sale, backorder cost, loss of goodwill	Decreases as Q increases (less chance of running out)
Purchase / Unit Cost (Cp)	The price paid per unit of the item itself	Usually constant, unless quantity discounts apply

The basic idea of every inventory model in this unit is this: ordering cost pulls us toward ordering in large quantities (fewer orders), while carrying cost pulls us toward ordering in small quantities (less stock sitting idle). The 'economic' order quantity is simply the quantity at which these two opposing pulls are exactly balanced, so total cost is minimised.

Note: Carrying cost Ch is often expressed as a percentage of unit cost ($i \times Cp$), where i is the annual carrying-cost rate (e.g., 20% per year). Either form may be used in numericals.

5.3 Economic Order Quantity (EOQ)

The Economic Order Quantity (EOQ) is the order quantity that minimises the total annual inventory cost, which is the sum of annual ordering cost and annual carrying cost. This is the single most important formula in inventory control, first derived by Ford W. Harris in 1913, and is therefore sometimes called the Harris-Wilson formula.

5.3.1 Assumptions of the Basic EOQ Model

- Demand rate (D) is known and constant throughout the year.
- Lead time (time between placing an order and receiving it) is known and constant.
- Replenishment is instantaneous — the entire order quantity arrives at once.
- No shortages are allowed (demand is always met on time).
- Ordering cost (C_o) and carrying cost (Ch) per unit per year are constant and known.
- Price per unit does not depend on order quantity (no quantity discounts) in the basic model.

5.3.2 Notation Used

Symbol	Meaning
D	Annual demand (units/year)
C_o	Ordering cost per order (₹/order)
Ch	Carrying (holding) cost per unit per year (₹/unit/year)
Q	Order quantity (units) — the decision variable

Symbol	Meaning
Q^*	Economic Order Quantity — the optimum value of Q
N	Number of orders per year = D/Q
t	Cycle time between orders = Q/D (in years)
TC	Total annual relevant cost

5.3.3 Deriving the EOQ Formula

Annual Ordering Cost = (Number of orders per year) $\times$ (Cost per order) = $(D/Q) \times Co$.

Annual Carrying Cost = (Average inventory) $\times$ (Carrying cost per unit) = $(Q/2) \times Ch$, since average stock over a cycle is half of Q under instantaneous replenishment with no shortages.

Total Annual Cost: $TC(Q) = (D/Q) \times Co + (Q/2) \times Ch$

To minimise TC , we differentiate with respect to Q and set the result to zero:

$$d(TC)/dQ = -(D \times Co)/Q^2 + Ch/2 = 0 \Rightarrow Q^2 = 2DCo/Ch$$

Economic Order Quantity: $Q^* = \sqrt{(2DCo / Ch)}$

Minimum Total Cost: $TC^* = \sqrt{(2 \times D \times Co \times Ch)}$

Other useful derived quantities:

- Optimum number of orders per year: $N^* = D / Q^* = \sqrt{(D \times Ch / 2Co)}$
- Optimum cycle time: $t^* = Q^*/D = \sqrt{(2Co / (D \times Ch))}$ (years), multiply by 12 for months or 365 for days.

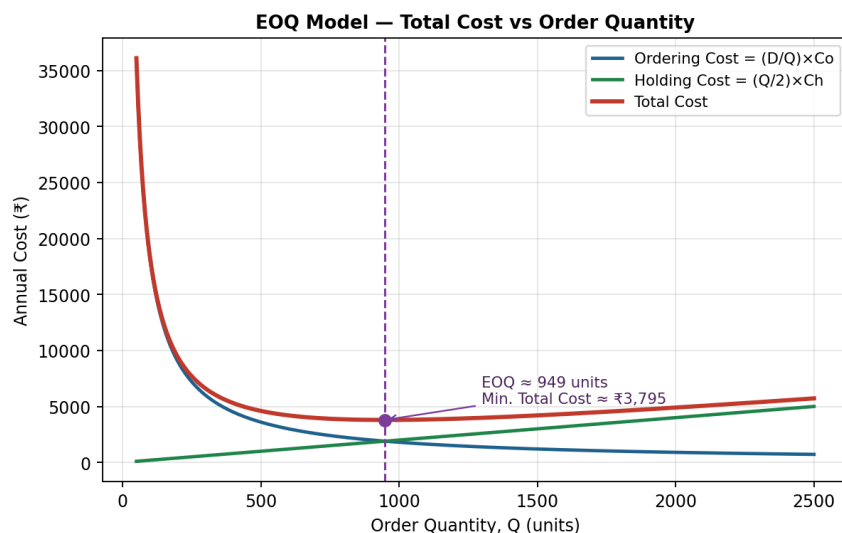


Fig. 5.2 — Ordering cost falls and carrying cost rises with Q ; total cost is minimised exactly where the two curves cross, at $Q = \text{EOQ}$.

Example 5.1 — Basic EOQ Calculation

A retailer sells 12,000 units of an item per year. The cost of placing one order is ₹150, and the carrying cost is ₹4 per unit per year. Find (a) the EOQ, (b) the minimum total annual cost, and (c) the optimum number of orders per year.

Solution:

Given: $D = 12,000$ units/year, $C_o = ₹150/\text{order}$, $C_h = ₹4/\text{unit/year}$.

(a) $Q^* = \sqrt{(2DC_o/C_h)} = \sqrt{(2 \times 12,000 \times 150 / 4)} = \sqrt{900,000} = 948.7 \approx 949$ units.

(b) $TC^* = \sqrt{(2 \times D \times C_o \times C_h)} = \sqrt{(2 \times 12,000 \times 150 \times 4)} = \sqrt{14,400,000} \approx ₹3,794.7 \approx ₹3,795$ per year.

(c) $N^* = D/Q^* = 12,000 / 948.7 \approx 12.65$ orders per year, i.e. an order roughly every $(365/12.65) \approx 29$ days.

Example 5.2 — EOQ with Carrying Cost Given as a Percentage of Price

A company requires 4,800 units of a component annually. The unit price is ₹20, ordering cost is ₹80 per order, and carrying cost is 25% of unit price per year. Determine the EOQ and the total minimum cost (including purchase cost).

Solution:

Given: $D = 4,800$ units/year, $C_p = ₹20/\text{unit}$, $C_o = ₹80/\text{order}$, $C_h = 25\% \times 20 = ₹5/\text{unit/year}$.

$Q^* = \sqrt{(2DC_o/C_h)} = \sqrt{(2 \times 4,800 \times 80 / 5)} = \sqrt{153,600} \approx 391.9 \approx 392$ units.

Minimum ordering + carrying cost: $TC^* = \sqrt{(2 \times D \times C_o \times C_h)} = \sqrt{(2 \times 4,800 \times 80 \times 5)} = \sqrt{3,840,000} \approx ₹1,959.6$.

Total cost including purchase price: $TC(\text{total}) = D \times C_p + TC^* = 4,800 \times 20 + 1,959.6 = 96,000 + 1,959.6 \approx ₹97,959.6$ per year.

Note: Adding the constant purchase-price term $D \times C_p$ does not change the value of Q^* — it only shifts the total-cost curve upward by a fixed amount, since this term does not depend on Q (in the basic model without quantity discounts).

5.4 Inventory Models with Deterministic Demand

The basic EOQ model assumed instantaneous replenishment and no shortages. Real situations are sometimes different — stock may arrive gradually (as in in-house production) or shortages may sometimes be deliberately allowed (as in backorder systems). Both situations still assume demand is known and constant (deterministic), but the cost formula changes slightly. We study two important deterministic models.

5.4.1 Model 1 — EOQ with Gradual Replenishment (Economic Production Quantity, EPQ)

This model fits a factory that manufactures its own stock rather than buying it. The production rate (p) is finite and stock builds up gradually while production is happening and demand is simultaneously consuming it; once production stops, stock depletes at the demand rate (d) until the next production run.

Since stock never builds up as fast as it would with instant delivery, the maximum inventory level is reduced by the factor $(1 - d/p)$. The formula becomes:

$Q^* = \sqrt{[2DCo / (Ch \times (1 - d/p))]}$, where d = annual demand rate, p = annual production rate (p must be greater than d).

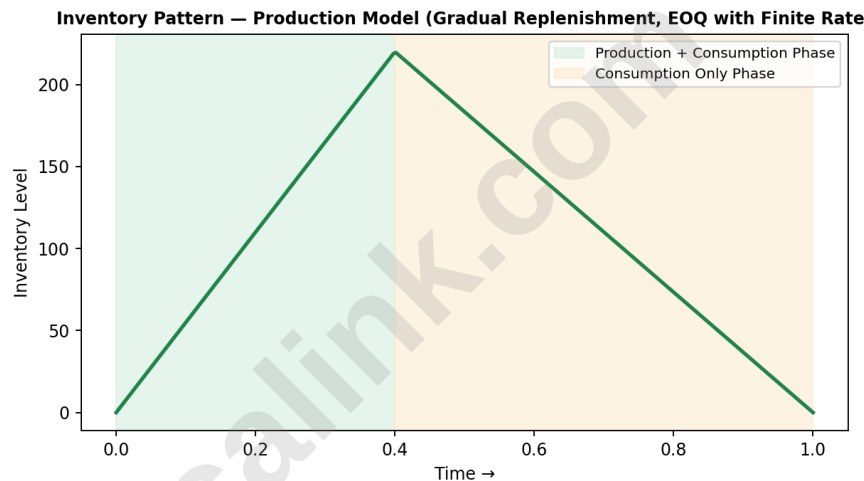


Fig. 5.3 — Inventory pattern under gradual replenishment: stock rises slowly during the production phase (production exceeds demand) and falls during the consumption-only phase.

Example 5.3 — Economic Production Quantity (EPQ)

A firm uses 7,200 units of an item annually, which it manufactures in-house at a rate of 24,000 units per year. Set-up cost per production run is ₹200, and carrying cost is ₹6 per unit per year. Find the economic production quantity.

Solution:

Given: $D (=d) = 7,200/\text{year}$, $p = 24,000/\text{year}$, $Co = ₹200$, $Ch = ₹6/\text{unit/year}$.

$$1 - d/p = 1 - (7,200/24,000) = 1 - 0.30 = 0.70.$$

$$Q^* = \sqrt{[2DCo / (Ch \times (1 - d/p))]} = \sqrt{[(2 \times 7,200 \times 200) / (6 \times 0.70)]} = \sqrt{(2,880,000 / 4.2)} = \sqrt{685,714.3} \approx 828.1 \approx 828 \text{ units per production run.}$$

5.4.2 Model 2 — EOQ with Planned Shortages (Backordering Allowed)

Sometimes a firm deliberately allows itself to run out of stock occasionally, because the shortage cost is lower than the extra carrying cost of always keeping more stock. Customers simply wait a short time for their order (a backorder). Here, a portion of the order cycle is spent in a 'stock-out' state.

Let S = maximum shortage (backorder) level allowed. The total cost now has three components: ordering, carrying (on the stock actually held), and shortage cost on the backordered units. Minimising total cost gives:

$$Q^* = \sqrt{[(2DC_o/Ch) \times ((Ch+C_s)/C_s)]} \text{ and } S^* = Q^* \times [C_h / (Ch+C_s)]$$

where C_s is the shortage cost per unit per year. Notice that as C_s becomes very large (shortages are very costly), the ratio $(Ch+C_s)/C_s$ approaches 1, and the formula correctly reduces to the basic EOQ formula with no shortages allowed.

Example 5.4 — EOQ with Planned Shortages

Annual demand for an item is 6,000 units. Ordering cost is ₹100 per order, carrying cost is ₹3 per unit per year, and shortage cost is ₹9 per unit per year. Find the optimum order quantity and the maximum shortage level.

Solution:

Given: $D = 6,000$, $C_o = ₹100$, $C_h = ₹3$, $C_s = ₹9$.

$$Q^* = \sqrt{[(2DC_o/Ch) \times ((Ch+C_s)/C_s)]} = \sqrt{[(2 \times 6,000 \times 100/3) \times (12/9)]} = \sqrt{[400,000 \times 1.333]} = \sqrt{533,333.3} \approx 730.3 \approx 730 \text{ units.}$$

$$S^* = Q^* \times C_h / (Ch+C_s) = 730.3 \times 3/12 = 730.3 \times 0.25 \approx 182.6 \approx 183 \text{ units.}$$

So the firm should order about 730 units each cycle, allowing the backorder level to build up to about 183 units before each new batch arrives.

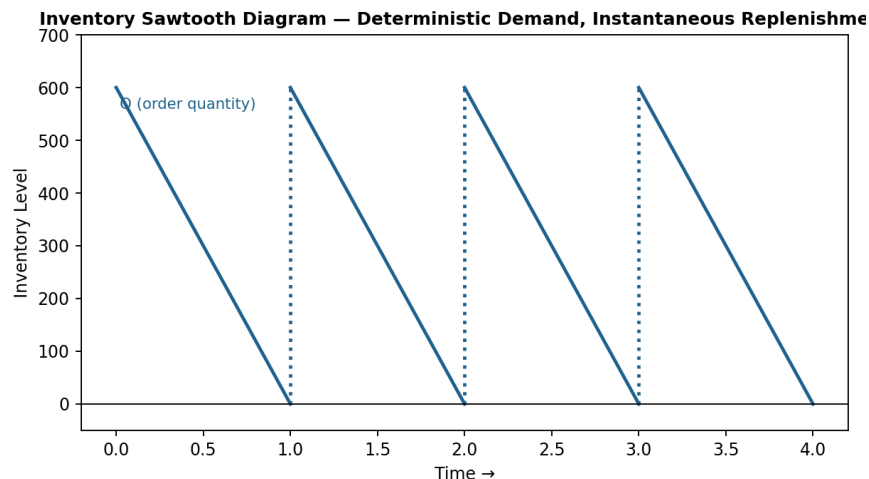


Fig. 5.4 — Basic sawtooth inventory pattern (instantaneous replenishment, no shortages) for comparison with the gradual-replenishment and shortage models above.

5.5 ABC Analysis

A typical factory store may carry thousands of different items, but giving every single item the same amount of attention and paperwork would be wasteful. ABC Analysis is a simple but powerful technique based on the Pareto Principle ('80-20 rule'): a small percentage of items usually accounts for a large percentage of total inventory value. ABC analysis classifies inventory items into three categories based on their annual consumption value (annual usage quantity $\times$ unit cost), so that management attention is focused where it matters most.

5.5.1 The Three Categories

Category	Typical % of Items	Typical % of Annual Value	Degree of Control
A — High Value	~10%	~70%	Tight control: frequent review, accurate records, low safety stock, EOQ calculated precisely
B — Moderate Value	~20%	~20%	Moderate control: periodic review, normal records
C — Low Value	~70%	~10%	Loose control: bulk ordering, large safety stock, simple/automatic reordering, infrequent review

5.5.2 Steps to Carry Out ABC Analysis

- Step 1: List every item along with its annual usage quantity and unit cost.
- Step 2: Compute the annual consumption value of each item = annual usage $\times$ unit cost.
- Step 3: Arrange items in descending order of annual consumption value.
- Step 4: Calculate cumulative value and cumulative % of total value, and cumulative % of total number of items.
- Step 5: Plot the cumulative % value against cumulative % items (a Pareto curve) and mark the boundaries where roughly 70% and 90% of cumulative value are reached.
- Step 6: Classify items above the first boundary as A, the next band as B, and the rest as C.

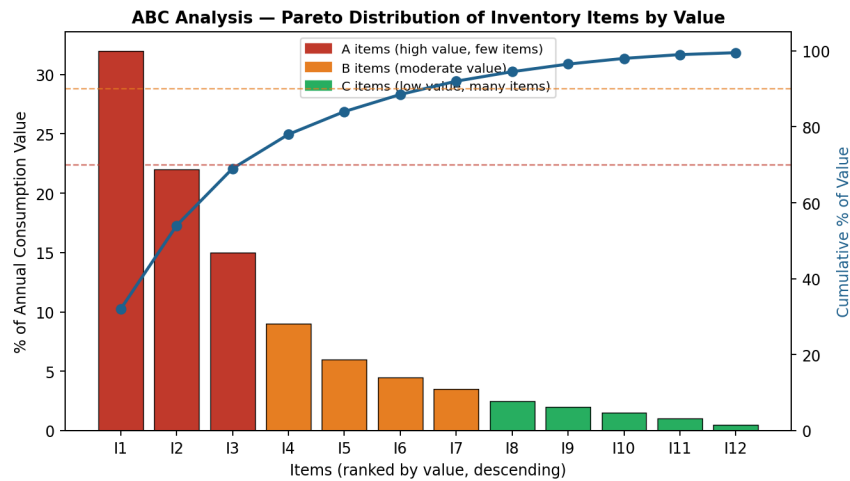


Fig. 5.5 — Pareto distribution of items by annual consumption value: a few items (A) contribute most of the value, while many items (C) contribute very little.

Example 5.5 — Performing ABC Analysis

A store has 5 items with the following annual usage and unit cost. Classify them using ABC analysis.

Item	Annual Usage (units)	Unit Cost (₹)
P	500	60
Q	4,000	2
R	200	150
S	1,000	8
T	100	5

Solution:

Step 1 — Annual consumption value = usage × unit cost:

P: $500 \times 60 = ₹30,000$. Q: $4,000 \times 2 = ₹8,000$. R: $200 \times 150 = ₹30,000$. S: $1,000 \times 8 = ₹8,000$. T: $100 \times 5 = ₹500$.

Total value = $30,000 + 8,000 + 30,000 + 8,000 + 500 = ₹76,500$.

Step 2 — Rank in descending order of value: P (30,000), R (30,000), Q (8,000), S (8,000), T (500).

Step 3 — Cumulative % of value: P = 39.2%, P+R = 78.4%, +Q = 88.9%, +S = 99.3%, +T = 100%.

Step 4 — Classification: A items = P, R (just 40% of items give ~78% of value); B items = Q, S (another 40% of items give ~21% of value); C items = T (remaining 20% of items give under 1% of value).

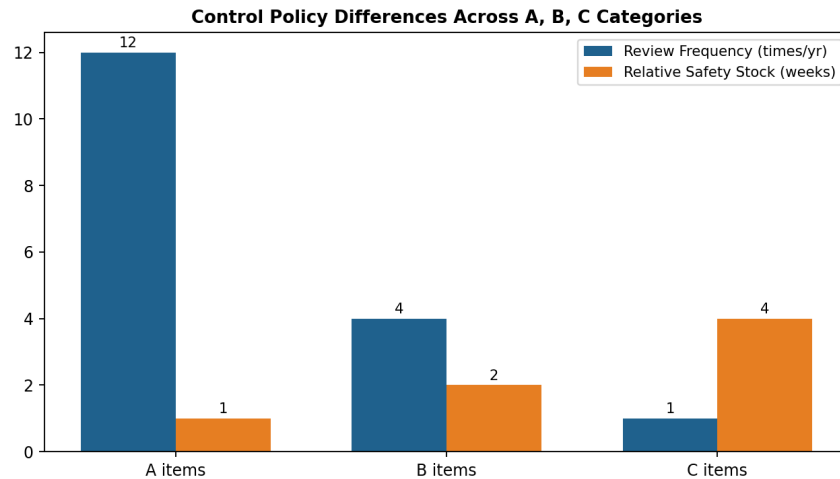


Fig. 5.6 — Illustrative difference in review frequency and safety stock policy applied across A, B, and C categories.

Note: ABC analysis is about value, not just usage quantity or just unit price alone — a cheap item used in huge quantities can still be an 'A' item, just like an expensive item used rarely. Always multiply usage × cost first.

Key Takeaways — Unit 5: Inventory Control

- ✓ Inventory exists to balance the cost of ordering against the cost of carrying stock; the right policy minimises their sum, not either one alone.
- ✓ Four key costs drive every model: ordering cost (C_o), carrying cost (C_h), shortage cost (C_s), and purchase cost (C_p).
- ✓ Basic EOQ: $Q^* = \sqrt{(2DC_o/C_h)}$, with minimum total cost $TC^* = \sqrt{(2DC_oC_h)}$.
- ✓ When stock is produced gradually rather than delivered instantly, EOQ is adjusted by the factor $(1 - d/p)$ to get the Economic Production Quantity.
- ✓ When planned shortages (backorders) are allowed, EOQ and the maximum shortage level S^* are found using the $(C_h + C_s)/C_s$ adjustment factor.
- ✓ ABC Analysis applies the 80-20 (Pareto) rule to classify items into A (high value, tight control), B (moderate), and C (low value, loose control), focusing management effort where it has the greatest financial impact.

— End of Unit 5 —