

Operations Research

Unit 1: Introduction to Operations Research

(5 Hours)

1. Introduction to Operations Research

Operations Research (OR) — also called Operational Research or Management Science — is a scientific discipline that applies advanced analytical methods to help make better decisions. It uses mathematical modelling, statistical analysis, and optimisation algorithms to arrive at optimal or near-optimal solutions to complex decision problems in business, industry, government, and the military.

The primary objective of OR is to replace intuitive decision-making with a more rational, quantitative approach, thereby reducing costs, increasing efficiency, and improving overall organisational performance.

Example 1.1 — Why OR Matters

A hospital has 10 operating theatres and must schedule 80 surgeries per week. Without OR, schedulers rely on experience and often leave theatres idle or create patient bottlenecks. By applying OR (specifically, integer linear programming), the hospital can optimise the schedule to maximise theatre utilisation, reduce patient waiting time, and cut overtime costs — all simultaneously.

2. History of Operations Research

The formal birth of Operations Research is generally traced to **World War II (1939–1945)**, when the British military faced the challenge of deploying limited radar resources most effectively against German air raids. A team of scientists led by **P. M. S. Blackett** at the Royal Air Force was assembled to study operational problems scientifically — this team is often credited as the first OR group.

Period	Key Development
Pre-1940s	Roots in scientific management (F. W. Taylor), statistics, and probability theory.
1939–1945	OR groups formed in UK & USA to solve military logistics, radar deployment, convoy routing, etc.

Period	Key Development
1947	George Dantzig develops the Simplex Method for Linear Programming — a landmark contribution.
1950s	OR techniques migrate to industry and business. PERT/CPM developed for project scheduling (Ford, 1958).
1960s–70s	Growth of computer power enables large-scale OR models. Integer programming, dynamic programming, and network flow problems become tractable.
1980s–90s	Heuristic and metaheuristic methods emerge (Simulated Annealing, Genetic Algorithms). Supply chain management models gain prominence.
2000s–present	Integration with Big Data, Machine Learning, and AI. Real-time optimisation in e-commerce, risk management, and logistics.

3. Stages of Development of Operations Research

The development of an OR solution follows a structured, iterative process. The six major stages are:

Stage 1: Problem Formulation

The real-world problem is identified, defined, and stated clearly. Objectives, constraints, and decision variables are specified. This is the most critical stage — a poorly defined problem leads to an irrelevant solution.

Stage 2: Mathematical Model Construction

The problem is translated into a mathematical model — equations and inequalities that represent the objective function and constraints. Assumptions are explicitly stated.

Stage 3: Solution of the Model

Appropriate mathematical techniques (Simplex method, dynamic programming, simulation, etc.) are applied to find the optimal or best feasible solution to the model.

Stage 4: Validation of the Model

The model is tested to ensure it accurately represents the real system. Sensitivity analysis is performed to examine how the solution changes with variations in parameters.

Stage 5: Implementation

The validated solution is translated back into actionable decisions and implemented in the real world. Clear communication with management and end-users is essential.

Stage 6: Monitoring and Control

The implemented solution is monitored over time. As conditions change, the model may need to be updated and re-solved.

Example 1.2 — Stages Applied to a Distribution Problem

Problem: A soft-drink company has 3 factories and 5 warehouses. It wants to minimise total transportation cost.

Stage 1: Identify factories (supply), warehouses (demand), unit costs.

Stage 2: Formulate as a Transportation LP model.

Stage 3: Apply Vogel's Approximation Method to get an initial solution; use MODI method for optimisation.

Stage 4: Check total cost against historical data; verify constraints are satisfied.

Stage 5: Present the optimal shipping schedule to the logistics team.

Stage 6: Review monthly as demand patterns change.

4. Relationship between Manager and OR Specialist

Successful OR projects depend on a productive partnership between the **manager** (who understands the business context) and the **OR specialist** (who understands the mathematical tools). Their roles and interactions can be summarised as follows:

Manager	OR Specialist
Defines business objectives and constraints	Translates objectives into mathematical models
Provides real-world data and domain knowledge	Selects and applies appropriate OR techniques
Makes final decisions based on OR recommendations	Interprets results and communicates implications clearly
Evaluates feasibility of proposed solutions	Performs sensitivity analysis and scenario testing
Allocates resources for model implementation	Documents models for future updates and reuse

Note: The relationship is collaborative, not hierarchical. The OR specialist advises; the manager decides. Miscommunication at any stage can lead to models that are mathematically correct but practically useless.

5. OR Tools and Techniques

Operations Research employs a rich toolkit. The major categories are:

- **Linear Programming (LP):** Optimises a linear objective function subject to linear constraints. Used in production planning, diet problems, and resource allocation.
- **Integer Programming (IP):** A form of LP where some or all decision variables must take integer values. Used in scheduling, capital budgeting, and network design.
- **Transportation and Assignment Models:** Special LP structures for distributing goods from sources to destinations (transportation) and matching agents to tasks (assignment).
- **Network Models:** Represent flows in networks — shortest path, maximum flow, minimum spanning tree, and PERT/CPM for project scheduling.
- **Dynamic Programming:** Solves multi-stage decision problems by breaking them into simpler sub-problems (Bellman's principle of optimality).
- **Queuing Theory:** Analyses waiting-line systems to balance service capacity against congestion costs. Used in banking, telecommunications, and healthcare.
- **Inventory Models:** Determine optimal order quantities and reorder points to minimise total inventory costs (EOQ model, ABC analysis).
- **Simulation:** Builds a computerised replica of a system and runs experiments to evaluate performance under various conditions — used when analytical models are intractable.
- **Game Theory:** Studies strategic interactions between rational decision-makers. Used in competitive bidding, pricing strategy, and military applications.
- **Replacement Theory:** Analyses optimal timing for replacing machinery or equipment as it deteriorates or becomes obsolete.

6. Applications of Operations Research

Sector	Typical OR Application
Manufacturing	Production scheduling, machine allocation, quality control
Transportation & Logistics	Vehicle routing, fleet management, supply chain optimisation
Finance & Banking	Portfolio optimisation, credit risk assessment, ATM cash management
Healthcare	Staff scheduling, operating theatre allocation, ambulance routing

Sector	Typical OR Application
Telecommunications	Network capacity planning, call centre staffing, spectrum allocation
Agriculture	Crop rotation planning, irrigation scheduling, livestock feed formulation
Military & Defence	Logistics, weapon system allocation, convoy planning
Retail & E-commerce	Inventory replenishment, warehouse layout, demand forecasting
Energy	Power generation scheduling, pipeline flow optimisation, load forecasting
Project Management	PERT/CPM for scheduling, resource levelling, cost–time trade-offs

Example 1.3 — OR in Airline Industry

Airlines use OR to solve one of the most complex scheduling problems in industry:

Fleet Assignment: Which aircraft type should fly each route to maximise profit given seat capacity, fuel cost, and demand?

Crew Scheduling: Assign pilots and cabin crew to flights satisfying union rules, rest requirements, and base constraints — an integer programming problem with millions of variables.

Revenue Management: Dynamic pricing of seats using LP and stochastic models to maximise revenue across different fare classes.

These OR applications save major airlines hundreds of millions of dollars annually.

7. Limitations of Operations Research

Despite its power, OR has several important limitations that practitioners must keep in mind:

- **Model vs. Reality Gap:** Mathematical models are simplifications of reality. Important real-world factors (human behaviour, political considerations, ethical concerns) may be difficult or impossible to quantify.
- **Data Requirements:** OR models require accurate, reliable, and often large volumes of data. Poor data quality ('garbage in, garbage out') leads to misleading results.
- **Computational Complexity:** Many OR problems (especially integer and combinatorial problems) are NP-hard — exact optimal solutions may be computationally infeasible for large instances.

- **Assumption of Linearity:** Classic LP assumes linear relationships between variables, which may not hold in practice (e.g., economies or diseconomies of scale).
- **Single-Objective Focus:** Most basic OR models optimise a single objective. Real decisions often involve multiple, conflicting objectives (cost, quality, time, environmental impact).
- **Implementation Resistance:** Managers and employees may resist OR recommendations due to lack of understanding, distrust of 'black-box' models, or organisational inertia.
- **Dynamic Environments:** A model solved today may become obsolete tomorrow if market conditions, technology, or regulations change rapidly.
- **Specialised Skills Required:** Building and solving OR models requires expertise in mathematics, statistics, and computing — skills that may not always be available in an organisation.

Unit 1 — Key Takeaways

- OR is a scientific, quantitative approach to decision-making originating in World War II.
- The six stages of OR are: problem formulation → modelling → solution → validation → implementation → monitoring.
- The manager–OR specialist relationship is collaborative; the manager decides, the specialist advises.
- Major OR tools include LP, transportation/assignment models, queuing theory, inventory models, simulation, and game theory.
- OR is applied across manufacturing, logistics, healthcare, finance, energy, and many other sectors.
- Limitations include model simplification, data dependency, computational complexity, and implementation challenges.